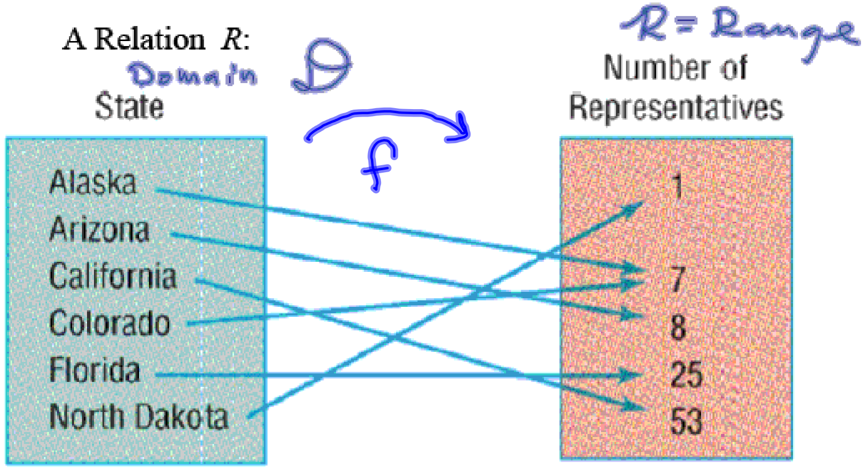
Section 1.1 Functions
Objectives:
- • What's a relation and when is it a function?
- • Find the value of a function.
- • Find the domain of a function
- • Form the sum, difference, product, and quotient of two functions. Also the difference quotient (Average slope)

Next time §1.1 is due. Plain white paper is best. ONE-SIDE ONLY

Leave margin @ Top.

Name, §1.1 MAT 121 # on Back.

Terms
- Relation
- Function
- Value of a function f at the number x. Evaluate
- Independent and dependent variables
- Domain
- Range
- Implicit Function *
- Argument Input



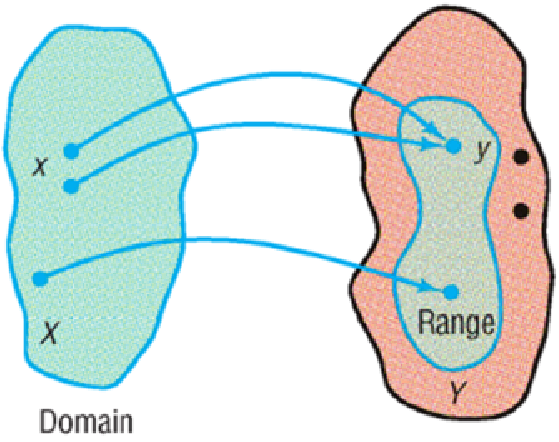
Is a function

$\mathcal{D}$ = {Arizona, Alaska, ..., North Dakota}

$\mathcal{R}$ = {1, 7, 8, 25, 53}

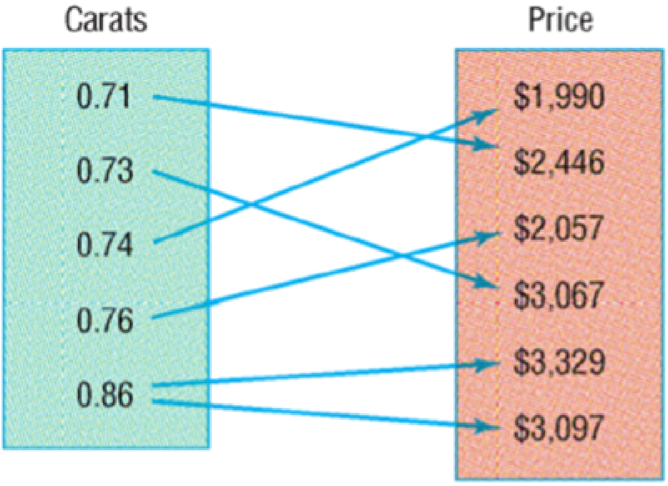
Let X and Y be two nonempty sets.* A **function** from X into Y is a relation that associates with each element of X exactly one element of Y.

The previous map was a function. Here's another:



No one bullet can hit more than one target.
No magic bullets

Is not 1-to-1



Not a function.
.86 corresponds to two values in the range.

Determine whether each relation represents a function.
If it is a function, state the domain and range.

(x, y)

{(2, 3), (4, 1), (3, -2), (2, -1)}

2 is mapped to 3 & -1.

{(-2, 3), (4, 1), (3, -2), (2, -1)}

Yes. $\mathcal{D} = \{-2, 4, 3, 2\}$

$\mathcal{R} = \{3, 1, -2, -1\}$

{(2, 3), (4, 3), (3, 3), (2, -1)}

No...

State whether this is a function.

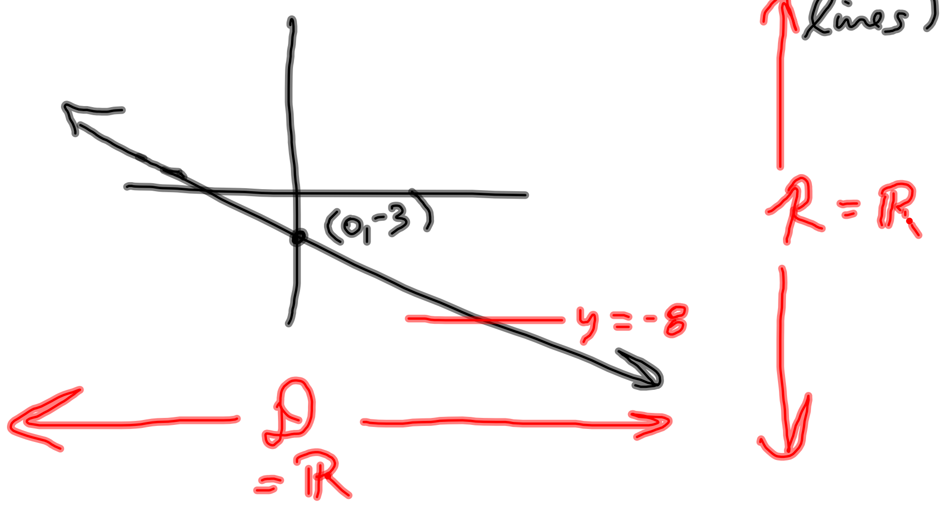
$$y = -\frac{1}{2}x - 3 \checkmark$$

$$y = \frac{\pm\sqrt{x^2+3} - 25x^3}{x^2 - 7x + 1}$$

is a linear function.

The domain of a line is all real #s, $\mathbb{R}$

$\therefore$ range of a line is $\mathbb{R}$ (except horizontal lines)



The functions with which we are concerned are real-valued functions with real-number inputs (or arguments). The argument x is real, and the corresponding value y is real. The correspondence between x and y is typically described by some combination of arithmetic operations on x, for instance,

argument

$$f(x) = -3x^2 + 2x,$$

All arithmetic.

Finding the value of a function means "evaluate":

For the function f defined by $f(x) = -3x^2 + 2x$, evaluate:

(a) $f(3)$ (b) $f(x) + f(3)$ (c) $f(-x)$

(d) $-f(x)$ (e) $f(x+3)$ (f) $\dfrac{f(x+h) - f(x)}{h}$, $h \neq 0$

(a) $f(3) = -3(3)^2 + 2(3)$
$= -3(9) + 6$
$= -27 + 6$
$= \boxed{-21 = f(3)}$

Different!

(b) $f(x) + f(3) = -3x^2 + 2x - 21$

(e) $f(x+3) = -3(x+3)^2 + 2(x+3)$
$= -3(x^2 + 6x + 9) + 2x + 6$

$(x+b)^2 = x^2 + 2bx + b^2$

$f(x) = -3x^2 + 2x$

$= -3x^2 - 18x - 27 + 2x + 6$
$= -3x^2 - 16x - 21$

$(x+c)^3 = x^3 + 3x^2c + 3xc^2 + c^3$

1 3 3 1

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

1 5 10 10 5 1

For the function f defined by $f(x) = -3x^2 + 2x$, evaluate:

(a) $f(3)$ (b) $f(x) + f(3)$ (c) $f(-x)$

(d) $-f(x)$ (e) $f(x+3)$ (f) $\dfrac{f(x+h) - f(x)}{h}$, $h \neq 0$

$$\frac{y_2 - y_1}{x_2 - x_1} \qquad x + h - x$$

$$= \frac{f(x+h) - f(x)}{x+h-x} = \frac{f(x+h) - f(x)}{h}$$ is the slope of the line connecting the two points on the graph of $f(x)$:

The difference quotient is the AVERAGE SLOPE of $f(x)$ between x & $x+h$.

$$-3x^2 + 2x = f(x)$$

$$f(\square) = -3\square^2 + 2\square$$

$$f(x+h) = -3(x+h)^2 + 2(x+h)$$
$$= -3(x^2 + 2xh + h^2) + 2x + 2h$$
$$= -3x^2 - 6xh - 3h^2 + 2x + 2h$$

Difference Quotient:

$$\frac{f(x+h) - f(x)}{h} = \frac{-3x^2 - 6xh - 3h^2 + 2x + 2h - (-3x^2 + 2x)}{h}$$

$$= \frac{-3x^2 - 6xh - 3h^2 + 2x + 2h + 3x^2 - 2x}{h}$$

$$= \frac{-6xh - 3h^2 + 2h}{h} = \frac{\cancel{h}(-6x - 3h + 2)}{\cancel{h}}$$

$$\boxed{= -6x - 3h + 2} \xrightarrow{h \to 0} -6x + 2$$

STOP! Calculus!

Determine if the equation $x = 2y^2 + 1$ defines y as a function of x.

General method.

Isolate y!

$2y^2 + 1 = x$

$2y^2 = x - 1$

$y^2 = \frac{x-1}{2}$

$y = \pm\sqrt{\frac{x-1}{2}}$ Not a function.

Two y-values associated with one x-value.

$x = 0$

$2y^2 + 1 = 0$

$\boxed{2y^2 = -1}$ oops!

Leads nowhere $x = 0$ isn't in the relation.

$y^2 = -\frac{1}{2}$

$x = 2$

$2y^2 + 1 = 2$

$2y^2 = 1$

$y^2 = \frac{1}{2}$

$y = \pm\sqrt{\frac{1}{2}}$

The y^2 is a problem

$2y^2 = 1 - x^2$

$y^2 = 1 - x^2$

$y = \pm\sqrt{1-x^2}$

Not a function.

The "±" creates 2 y-values for several x-values e.g.

$x = 0 \begin{cases} y = 1 \\ y = -1 \end{cases}$

$x = \sqrt{\frac{3}{4}}$

$y = \pm\sqrt{1 - \frac{3}{4}} = \pm\sqrt{\frac{1}{4}} = \pm\frac{1}{2}$

$x = \frac{\sqrt{3}}{2} \begin{cases} \frac{1}{2} = y \\ -\frac{1}{2} = y \end{cases}$

Aug 31-8:37 AM

Summary

Important Facts About Functions

(a) For each x in the domain of f, there is exactly one image $f(x)$ in the range; however, an element in the range can result from more than one x in the domain.

(b) f is the symbol that we use to denote the function. It is symbolic of the equation that we use to get from an x in the domain to $f(x)$ in the range.

(c) If $y = f(x)$, then x is called the independent variable or argument of f, and y is called the dependent variable or the value of f at x.